Correspondence

"Gap Effect" in Measurement of Large Permittivities

Complex permittivity is frequently determined from measurements of the transmission—or reflection coefficient of the dominant mode of a uniformly-filled waveguide section [1]. Uniformity is difficult to achieve in practice, however, because of unavoidable gaps between the sample and the waveguide walls. The importance of gaps increases with increasing magnitude of the complex permittivity and may cause large errors in measurements of high-permittivity or high-loss (e.g., semiconducting) dielectrics if proper corrections are not made. The purpose of this note is to examine the range of validity of several correction formulas found in the literature by comparing their predictions of measured permittivity with values actually measured with germanium. During these measurements, the true magnitude of the complex relative permittivity of the germanium was varied between 16.4 and 860.0 by varying the sample temperature.

An empirical formula to correct for the air gap has been given by Redheffer [2]. This formula can be written

$$\{\epsilon_r(m) - 1\} = \{\epsilon_r - 1\}\{1 - t/b\} \qquad (1)$$

where $\epsilon_r(m)$ and ϵ_r are "measured" and "actual" relative permittivities, respectively, and b and t are dimensions defined in Fig. 1. Other correction formulas have been given by Westphal [3] and by Bussey and Gray [4]. Westphal's formula can be written [5]

$$\epsilon_r(m) = \frac{\epsilon_r}{1 + \{\epsilon_r - 1\}\{t/b\}} \qquad (2)$$

while the simple perturbation formula of Bussey and Gray (evaluated, in this case, for the rectangular TE_{10} mode) is

$$\epsilon_r(m) = \epsilon_r\{1 - [\epsilon_r - 1][t/b]\}. \qquad (3)$$

Equation (3) is recognized as being the first two terms of the expansion of (2) in a power series in $[\epsilon_r - 1][t/b]$.

If one assumes that ϵ_r is purely real, one can plot (1)–(3) on a single graph. Such plots are shown in Fig. 2 for $[t/b] = 0.015$. Of the three equations, one sees that only (2) saturates for large ϵ_r, approaching the limiting value $[b/t]$ regardless of the permittivity of the material.

Experimental points have been plotted in Fig. 2 for comparison with the theoretical curves. The experimental values were obtained by using a transmission bridge at 10.5 Gc/s to measure the complex permittivity of a sample of intrinsic germanium whose height was about 150 microns less than the b-dimension of the waveguide ($[t/b] \cong 0.015$). During measurement, the sample temperature was varied between 300 and 440 degrees K. This varied the conductivity between 2 and 500 mho/m and allowed measurements of complex $\epsilon_r(m)$ to

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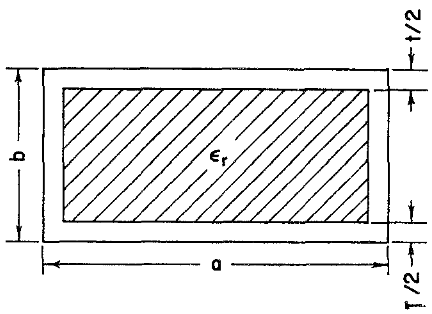

Fig. 1. Rectangular waveguide showing gap between dielectric sample and waveguide walls.

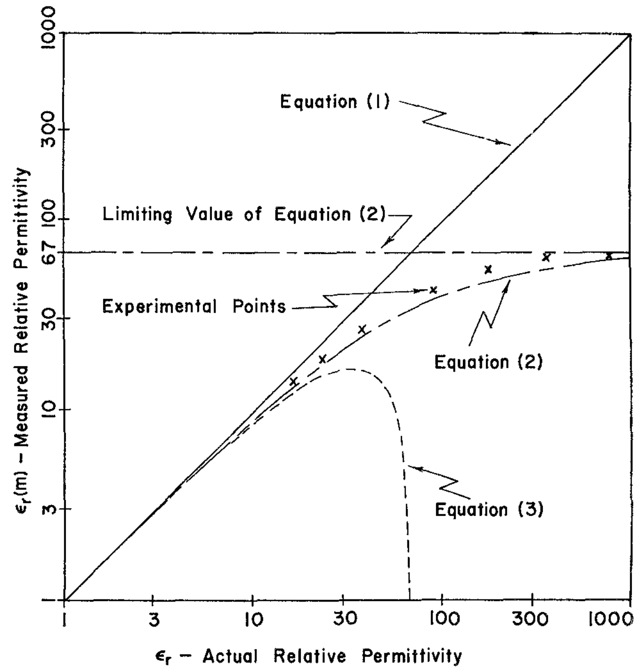

Fig. 2. Measured relative permittivity as function of actual relative permittivity of sample for $[t/b] = .015$. Theoretical curves assume permittivity is purely real, while experimental points are magnitudes of complex quantities.

be taken over the wide range of $|\epsilon_r|$ as shown in Fig. 2. For simplicity, the complex experimental data have been displayed in Fig. 2 by plotting the magnitude $|\epsilon_r(m)|$ as a function of $|\epsilon_r|$.

For a lossy material, (1)–(3) relate complex values of $\epsilon_r(m)$ to complex values of ϵ_r but do not give exact relationships between $|\epsilon_r(m)|$ and $|\epsilon_r|$ directly. They approach true relationships between complex permittivity *magnitudes* at low and high conductivities, however. Between these extremes, values of $|\epsilon_r(m)|$ determined by operating directly on $|\epsilon_r|$ with (2) are somewhat low, a maximum of about 30 percent low at $|\epsilon_r| = 200$ for the sample used in the present experiment. Thus, considering the approximation involved in comparing the theoretical curves and experimental points of Fig. 2, one sees that (2) agrees quite well with the measurements. Note that the experimental values approach $[b/t]$ for large $|\epsilon_r|$ as (2) predicts. The other two equations do not agree as well with the measurements although (3) is satisfactory for small $|(\epsilon_r - 1)(t/b)|$ (the region in which it was actually used by Bussey and Gray [4]).

Equation (2) can be given a simple physical interpretation [6]. If one equates the terminal impedances of the parallel plane capacitors of Fig. 3(a) and solves for $\epsilon_r(m)$, one obtains (2). Thus, (2) can be represented by the lumped-parameter circuit of Fig. 3(b). One sees that in this interpretation, the air gap merely introduces a spurious impedance in series with the bulk impedance of the material. The importance of the air gap is determined completely by the relative size of these two impedances.

Considering the complexity of a general treatment of an inhomogeneously filled waveguide, the simplicity of (2) and of the circuit of Fig. 3(b) is somewhat surprising. In particular, one might expect the sample dimension in the propagation direction to be an important consideration. The

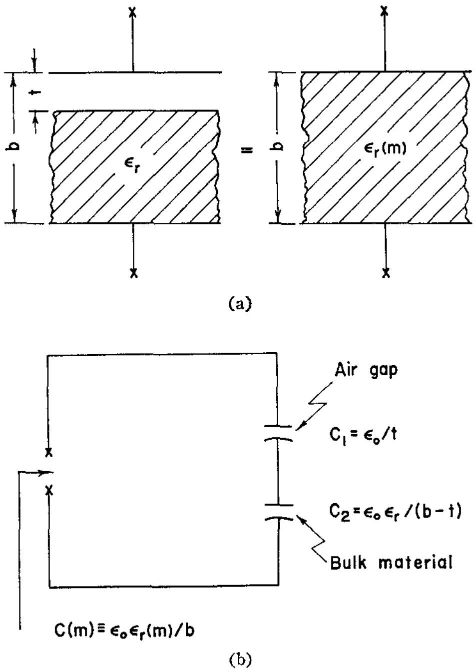

Fig. 3. Physical interpretation of Eq. (2). (a) Parallel plane model. (b) Equivalent circuit.

simplicity results from the fact that (2) considers the effect of the gap on only the dominant mode and neglects all higher-order modes generated at the planes of discontinuity between empty and "filled" waveguides, an approach that is rigorously justified for small perturbations only

$$(|\{\epsilon_r - 1\}\{t/b\}| \ll 1).$$

Since the maximum value of

$$|\{\epsilon_r - 1\}\{t/b\}|$$

was about 13 in the present experiment, one sees that (2) gives a "surprisingly" good qualitative description of "gap effect" for large perturbations. One should generally not expect it to be quantitatively accurate in this range, however.

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Fringing Capacitance in Strip-Line Coupler Design

Very useful relationships between strip-line directional coupler dimensions and the even- and odd-mode impedance have been derived by S. B. Cohn for both the case of side-by-side strips [1] (edge coupling) and broadside coupling [2]. In each case the relations for even- and odd-mode impedance contain, respectively, terms for even- and odd-mode fringing capacitance per unit length. Cohn has derived relationships for both even- and odd-mode fringing capacitances for the case of side-by-side strips [1] and broadside coupling [2] for strips of zero thickness, and has published a paper on thickness corrections [3]. Gunderson and Guida [4] have shown that for the broadside coupled case the even- and odd-mode fringing capacitances are not independent and have thus derived a relationship between the coupler dimensions and even and odd impedance which does not involve expressions for the fringing capacitances. This has led them to formulate a simpler design procedure [4].

The purpose of this communication is to show that such a relation also exists for side-by-side strips.

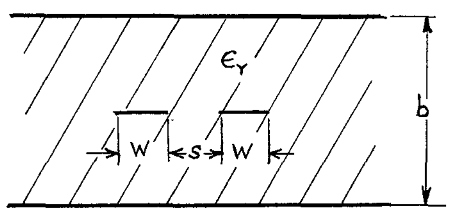

Fig. 1. Side-by-side strip coupler cross section.

Cohn's equations [1] (11) and (12) can be written [5] when $W/b \rightarrow 0.35$:

$$Z_1 = \frac{\eta_0}{4\sqrt{\epsilon_r}\left[\frac{W}{b} + \frac{C_f + C_{f1}}{2\epsilon_r\epsilon_0}\right]} \quad (\text{in } \Omega)$$

$$Z_2 = \frac{\eta_0}{4\sqrt{\epsilon_r}\left[\frac{W}{b} + \frac{C_f + C_{f2}}{2\epsilon_r\epsilon_0}\right]} \quad (\text{in } \Omega)$$

from which one can solve for

$$C_{f2} - C_{f1} = \frac{\eta_0\epsilon_0\sqrt{\epsilon_r}}{2}\left(\frac{1}{Z_2} - \frac{1}{Z_1}\right) \quad (\text{in F/m})$$

where:

Z_1 = odd characteristic impedance
Z_2 = even characteristic impedance
η_0 = intrinsic impedance of free space $= 377\ \Omega$
ϵ_0 = permittivity of free space $= 8.85 \times 10^{-12}$ *F/m*
ϵ_r = relative permittivity of strip-line dielectric
b = separation between ground planes
W = width of each conducting strip
C_f = fringing capacitance from outside edges of strip-to-ground planes (in *F/m*)
C_{f1} = fringing capacitance at adjacent edges of strips under odd-mode excitation (in *F/m*)
C_{f2} = fringing capacitance at adjacent edges of strips under even-mode excitation (in *F/m*).

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Also Cohn's equations [1] (13) and (14) can be solved to give

$$C_{f2} - C_{f1} = \epsilon_r\epsilon_0\frac{2}{\pi}\left[-\ln\left(\cosh\frac{\pi s}{2b}\right) + \ln\left(\sinh\frac{\pi s}{2b}\right)\right]$$

$$C_{f2} - C_{f1} = \epsilon_r\epsilon_0\frac{2}{\pi}\ln\left(\tanh\frac{\pi s}{2b}\right)$$

where:

s = separation between strips.

This can be equated to the relation for fringing capacitances given above to give

$$\frac{s}{b} = \frac{2}{\pi}\operatorname{arctanh}\left[\exp\frac{\eta_0}{4\sqrt{\epsilon_r}}\left(\frac{1}{Z_2} - \frac{1}{Z_1}\right)\right]$$

$$\frac{s}{b} = -\frac{1}{\pi}\ln\left[\tanh\frac{\pi\eta_0}{8\sqrt{\epsilon_r}}\left(\frac{1}{Z_1} - \frac{1}{Z_2}\right)\right].$$

Thus, the ratio of strip separation (between adjacent edges) to ground plane separation can be determined without the need to evaluate fringing capacitance. If desired this ratio can be expressed directly in terms of the midband voltage-coupling ratio k, since [4]

$$Z_1 = Z_0\sqrt{\left(\frac{1-k}{1+k}\right)}$$

where Z_0 = characteristic impedance

$$Z_2 = Z_0\sqrt{\left(\frac{1+k}{1-k}\right)}$$

$$\frac{s}{b} = -\frac{1}{\pi}\ln\left\{\tanh\left[\frac{\pi\eta_0}{4\sqrt{\epsilon_r}\,Z_0}\,\frac{k}{\sqrt{1-k^2}}\right]\right\}$$

and [1] for strips of zero thickness

$$\frac{W}{b} = \frac{\eta_0}{4\sqrt{\epsilon_r}\,Z_0}\sqrt{\left(\frac{1-k}{1+k}\right)} - \frac{s}{2b} + \frac{1}{\pi}\left[\ln\cosh\frac{\pi s}{2b} - \ln 2\right].$$

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